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Ref.: Physical chem-II, Unit-I, Quantum mechanics Dated: 15/04/2020

Linear operator:- An operator is a symbolic instruction for carrying out certain mathematical operations such as multiplication, differentiation, integration etc. on an operand that follows the operator. An operand is usually a mathematical function. Acting on a function, an operator generates a new function. For Example -

(operator)	(operand)	= New function
x	y	$= xy$
d/dx	x^2	$= 2x$
$\int(dx)$	x^2	$= x^3/3 +$ constant

An operator is represented by placing a caret ($\wedge$) symbol over a Capital English letter. i.e an operator $\hat{A}$ is said to be linear when it satisfies the following relation.

$$\hat{A}[c_1 f_1(x) + c_2 f_2(x)] = c_1 \hat{A} f_1(x) + c_2 \hat{A} f_2(x) \quad \text{---(1)}$$

where c_1, c_2 are real or complex constants.

$f_1(x)$ & $f_2(x)$ are functions of x.

It is easy to verify that multiplication, differential and integral operators are linear.

$$\frac{d}{dx}(x^2 + 2x) = \frac{d}{dx}x^2 + 2\frac{d}{dx}x = 2x + 2$$

$$\int (x^2 + 2x)dx = \int x^2 dx + 2\int x\,dx = \frac{x^3}{3} + x^2 + \text{constant}$$

$$\sqrt{9+16} = 5 \neq (\sqrt{9} + \sqrt{16}) = 7$$

$$\log(x+y) \neq \log x + \log y$$

Two operators $\hat{A}$ & $\hat{B}$ are said to commute when they satisfy the following relation with reference to a function.

$$\hat{A}\hat{B}f = \hat{B}\hat{A}f$$

$$\text{or } (\hat{A}\hat{B} - \hat{B}\hat{A})f = [\hat{A}, \hat{B}]f = 0 \quad \text{---(2)}$$

where $[\hat{A}, \hat{B}]$ represents the commutator of $\hat{A}$ and $\hat{B}$.

The commutator of two operators is an operator.