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Hermitian operators:— An operator $\hat{A}$ is said to be Hermitian if it satisfies the following relation.

$$\int (\hat{A}\psi_i)^* \psi_j \, d\tau = \int \psi_i^* \hat{A} \psi_j \, d\tau \quad \text{——— (1)}$$

where ψ_i and ψ_j are two wave functions.

In bra-ket notation eqn (1) becomes.

$$\langle \hat{A}\psi_i | \psi_j \rangle = \langle \psi_i | \hat{A} | \psi_j \rangle \quad \text{——— (2)}$$

Eqn (1) and (2) is obviously satisfied for ~~two~~ any real multiplication operator.

For example.

if $\hat{A} = x$ then

$$\langle \hat{A}\psi_i | \psi_j \rangle = \langle x\psi_i | \psi_j \rangle = \int (x\psi_i)^* \psi_j \, dx$$

$$= \int \psi_i^* x \psi_j \, dx = \langle \psi_i | x | \psi_j \rangle$$

Thus $A = x$ is Hermitian

It is very important to note that a quantum mechanical operator satisfies the following condition known as Hermitian condition. If an operator $\hat{A}$ has two eigenfunctions ψ & ϕ and if,

$$\int \psi (\hat{A}\phi) \, d\tau = \int (\hat{A}\psi) \phi \cdot d\tau \quad \text{——— (1)}$$

when ψ and ϕ are real

$$\text{or} \int \psi^* (\hat{A}\phi) \, d\tau = \int (\hat{A}\psi)^* \cdot \phi \, d\tau \quad \text{——— (2)}$$

when ψ and ϕ are complex, ψ^* is the complex conjugate of ψ and $d\tau$ is the volume element of space in which the function is defined then the operator $\hat{A}$ is called Hermitian operator.

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